

Trigonometry Handout

Definition of the Trig Functions

Right triangle definition

For this definition we assume that

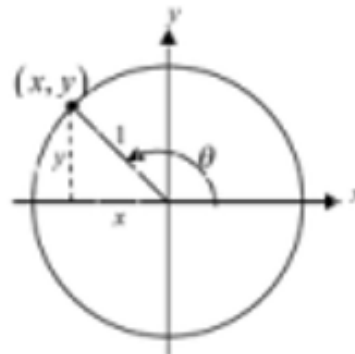
$$0 < \theta < \frac{\pi}{2} \text{ or } 0^\circ < \theta < 90^\circ.$$



$$\begin{aligned} \sin \theta &= \frac{\text{opposite}}{\text{hypotenuse}} & \csc \theta &= \frac{\text{hypotenuse}}{\text{opposite}} \\ \cos \theta &= \frac{\text{adjacent}}{\text{hypotenuse}} & \sec \theta &= \frac{\text{hypotenuse}}{\text{adjacent}} \\ \tan \theta &= \frac{\text{opposite}}{\text{adjacent}} & \cot \theta &= \frac{\text{adjacent}}{\text{opposite}} \end{aligned}$$

Unit circle definition

For this definition θ is any angle.



$$\begin{aligned} \sin \theta &= \frac{y}{1} = y & \csc \theta &= \frac{1}{y} \\ \cos \theta &= \frac{x}{1} = x & \sec \theta &= \frac{1}{x} \\ \tan \theta &= \frac{y}{x} & \cot \theta &= \frac{x}{y} \end{aligned}$$

Facts and Properties

Domain

The domain is all the values of θ that can be plugged into the function.

$$\begin{aligned} \sin \theta, \quad \theta &\text{ can be any angle} \\ \cos \theta, \quad \theta &\text{ can be any angle} \\ \tan \theta, \quad \theta &\neq \left(n + \frac{1}{2}\right)\pi, \quad n = 0, \pm 1, \pm 2, \dots \\ \csc \theta, \quad \theta &\neq n\pi, \quad n = 0, \pm 1, \pm 2, \dots \\ \sec \theta, \quad \theta &\neq \left(n + \frac{1}{2}\right)\pi, \quad n = 0, \pm 1, \pm 2, \dots \\ \cot \theta, \quad \theta &\neq n\pi, \quad n = 0, \pm 1, \pm 2, \dots \end{aligned}$$

Range

The range is all possible values to get out of the function.

$$\begin{aligned} -1 \leq \sin \theta \leq 1 & \quad \csc \theta \geq 1 \text{ and } \csc \theta \leq -1 \\ -1 \leq \cos \theta \leq 1 & \quad \sec \theta \geq 1 \text{ and } \sec \theta \leq -1 \\ -\infty < \tan \theta < \infty & \quad -\infty < \cot \theta < \infty \end{aligned}$$

Period

The period of a function is the number, T , such that $f(\theta + T) = f(\theta)$. So, if ω is a fixed number and θ is any angle we have the following periods.

$$\begin{aligned} \sin(\omega\theta) &\rightarrow T = \frac{2\pi}{\omega} \\ \cos(\omega\theta) &\rightarrow T = \frac{2\pi}{\omega} \\ \tan(\omega\theta) &\rightarrow T = \frac{\pi}{\omega} \\ \csc(\omega\theta) &\rightarrow T = \frac{2\pi}{\omega} \\ \sec(\omega\theta) &\rightarrow T = \frac{2\pi}{\omega} \\ \cot(\omega\theta) &\rightarrow T = \frac{\pi}{\omega} \end{aligned}$$

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Formulas and Identities

Tangent and Cotangent Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

Reciprocal Identities

$$\csc \theta = \frac{1}{\sin \theta} \quad \sin \theta = \frac{1}{\csc \theta}$$

$$\sec \theta = \frac{1}{\cos \theta} \quad \cos \theta = \frac{1}{\sec \theta}$$

$$\cot \theta = \frac{1}{\tan \theta} \quad \tan \theta = \frac{1}{\cot \theta}$$

Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

Even/Odd Formulas

$$\sin(-\theta) = -\sin \theta \quad \csc(-\theta) = -\csc \theta$$

$$\cos(-\theta) = \cos \theta \quad \sec(-\theta) = \sec \theta$$

$$\tan(-\theta) = -\tan \theta \quad \cot(-\theta) = -\cot \theta$$

Periodic Formulas

If n is an integer.

$$\sin(\theta + 2\pi n) = \sin \theta \quad \csc(\theta + 2\pi n) = \csc \theta$$

$$\cos(\theta + 2\pi n) = \cos \theta \quad \sec(\theta + 2\pi n) = \sec \theta$$

$$\tan(\theta + \pi n) = \tan \theta \quad \cot(\theta + \pi n) = \cot \theta$$

Double Angle Formulas

$$\sin(2\theta) = 2 \sin \theta \cos \theta$$

$$\cos(2\theta) = \cos^2 \theta - \sin^2 \theta$$

$$= 2 \cos^2 \theta - 1$$

$$= 1 - 2 \sin^2 \theta$$

$$\tan(2\theta) = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

Degrees to Radians Formulas

If x is an angle in degrees and t is an angle in radians then

$$\frac{\pi}{180} = \frac{t}{x} \quad \Rightarrow \quad t = \frac{\pi x}{180} \quad \text{and} \quad x = \frac{180t}{\pi}$$

Half Angle Formulas

$$\sin^2 \theta = \frac{1}{2}(1 - \cos(2\theta))$$

$$\cos^2 \theta = \frac{1}{2}(1 + \cos(2\theta))$$

$$\tan^2 \theta = \frac{1 - \cos(2\theta)}{1 + \cos(2\theta)}$$

Sum and Difference Formulas

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$

$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}$$

Product to Sum Formulas

$$\sin \alpha \sin \beta = \frac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$

$$\cos \alpha \cos \beta = \frac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)]$$

$$\sin \alpha \cos \beta = \frac{1}{2}[\sin(\alpha + \beta) + \sin(\alpha - \beta)]$$

$$\cos \alpha \sin \beta = \frac{1}{2}[\sin(\alpha + \beta) - \sin(\alpha - \beta)]$$

Sum to Product Formulas

$$\sin \alpha + \sin \beta = 2 \sin\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right)$$

$$\sin \alpha - \sin \beta = 2 \cos\left(\frac{\alpha + \beta}{2}\right) \sin\left(\frac{\alpha - \beta}{2}\right)$$

$$\cos \alpha + \cos \beta = 2 \cos\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right)$$

$$\cos \alpha - \cos \beta = -2 \sin\left(\frac{\alpha + \beta}{2}\right) \sin\left(\frac{\alpha - \beta}{2}\right)$$

Cofunction Formulas

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta \quad \cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$$

$$\csc\left(\frac{\pi}{2} - \theta\right) = \sec \theta \quad \sec\left(\frac{\pi}{2} - \theta\right) = \csc \theta$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot \theta \quad \cot\left(\frac{\pi}{2} - \theta\right) = \tan \theta$$

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Special Triangles

